ADVANCED SUBSIDIARY GCE UNIT

4752/01

MATHEMATICS (MEI)

Concepts for Advanced Mathematics (C2)

THURSDAY 7 JUNE 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

[Turn over

Section A (36 marks)

1 **(i)** State the exact value of $\tan 300°$. [1]

(ii) Express 300° in radians, giving your answer in the form $k\pi$, where k is a fraction in its lowest terms. [2]

2 Given that $y = 6x^{\frac{3}{2}}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Show, without using a calculator, that when $x = 36$ the value of $\frac{d^2y}{dx^2}$ is $\frac{3}{4}$. [5]

3

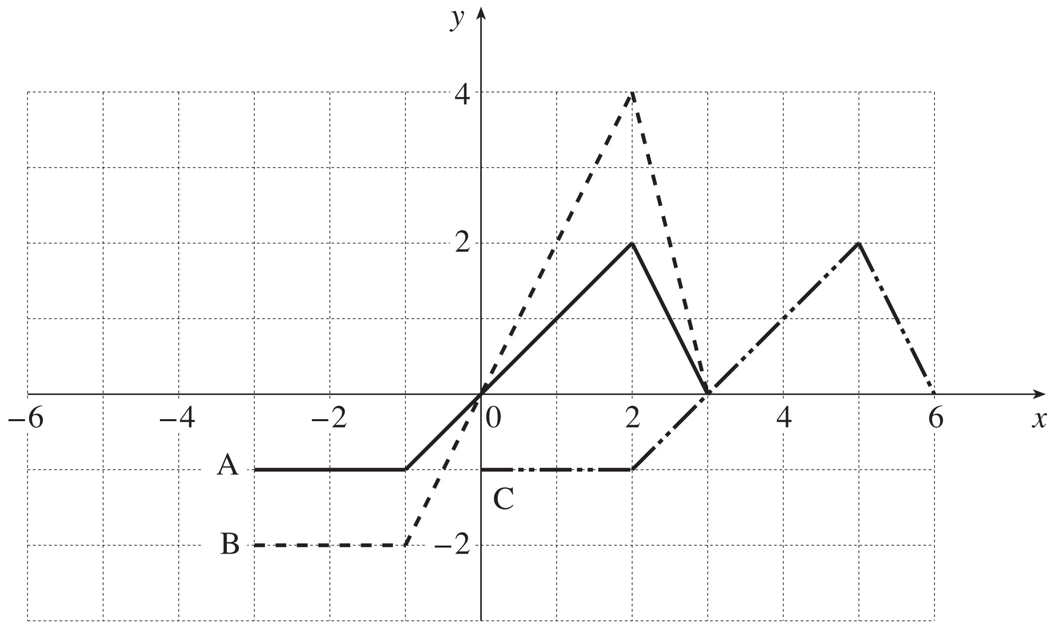

Fig. 3

Fig. 3 shows sketches of three graphs, A, B and C. The equation of graph A is $y = \mathrm{f}(x)$.

State the equation of

(i) graph B, [2]

(ii) graph C. [2]

4 **(i)** Find the second and third terms of the sequence defined by the following.

$$t_{n+1} = 2t_n + 5$$
$$t_1 = 3$$ [2]

(ii) Find $\sum_{k=1}^{3} k(k+1)$. [2]

5 A sector of a circle of radius 5 cm has area 9 cm^2.

Find, in radians, the angle of the sector.

Find also the perimeter of the sector. [5]

6 **(i)** Write down the values of $\log_a 1$ and $\log_a a$, where $a > 1$. [2]

(ii) Show that $\log_a x^{10} - 2\log_a\left(\frac{x^3}{4}\right) = 4\log_a(2x)$. [3]

7 **(i)** Sketch the graph of $y = 3^x$. [2]

(ii) Use logarithms to solve the equation $3^x = 20$. Give your answer correct to 2 decimal places. [3]

8 **(i)** Show that the equation $2\cos^2\theta + 7\sin\theta = 5$ may be written in the form

$$2\sin^2\theta - 7\sin\theta + 3 = 0.$$ [1]

(ii) By factorising this quadratic equation, solve the equation for values of θ between 0° and 180°. [4]

Section B (36 marks)

9 The equation of a cubic curve is $y = 2x^3 - 9x^2 + 12x - 2$.

(i) Find $\frac{dy}{dx}$ and show that the tangent to the curve when $x = 3$ passes through the point $(-1, -41)$. [5]

(ii) Use calculus to find the coordinates of the turning points of the curve. You need not distinguish between the maximum and minimum. [4]

(iii) Sketch the curve, given that the only real root of $2x^3 - 9x^2 + 12x - 2 = 0$ is $x = 0.2$ correct to 1 decimal place. [3]

10 Fig. 10 shows the speed of a car, in metres per second, during one minute, measured at 10-second intervals.

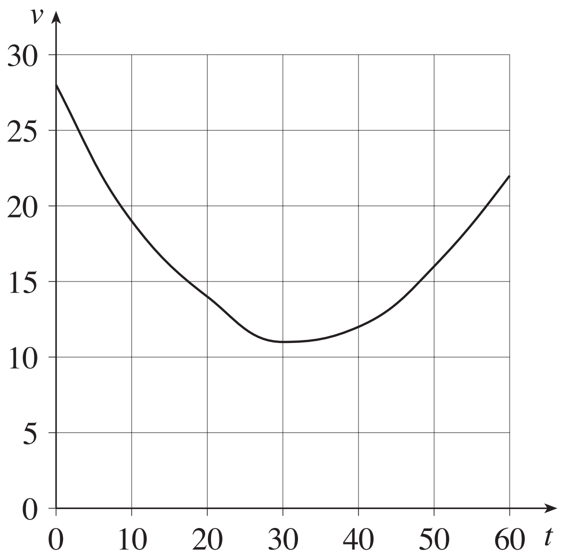

Fig. 10

The measured speeds are shown below.

Time (t seconds)	0	10	20	30	40	50	60
Speed ($v\,\mathrm{m\,s^{-1}}$)	28	19	14	11	12	16	22

(i) Use the trapezium rule with 6 strips to find an estimate of the area of the region bounded by the curve, the line $t = 60$ and the axes. [This area represents the distance travelled by the car.] [4]

(ii) Explain why your calculation in part **(i)** gives an overestimate for this area. Use appropriate rectangles to calculate an underestimate for this area. [3]

The speed of the car may be modelled by $v = 28 - t + 0.015t^2$.

(iii) Show that the difference between the value given by the model when $t = 10$ and the measured value is less than 3% of the measured value. [2]

(iv) According to this model, the distance travelled by the car is

$$\int_0^{60} (28 - t + 0.015t^2)\,dt.$$

Find this distance. [3]

11 **(a)** André is playing a game where he makes piles of counters. He puts 3 counters in the first pile. Each successive pile he makes has 2 more counters in it than the previous one.

(i) How many counters are there in his sixth pile? [1]

(ii) André makes ten piles of counters. How many counters has he used altogether? [2]

(b) In another game, played with an ordinary fair die and counters, Betty needs to throw a six to start.

The probability P_n of Betty starting on her nth throw is given by

$$P_n = \frac{1}{6} \times \left(\frac{5}{6}\right)^{n-1}.$$

(i) Calculate P_4. Give your answer as a fraction. [2]

(ii) The values $P_1, P_2, P_3, \ldots$ form an infinite geometric progression. State the first term and the common ratio of this progression.

Hence show that $P_1 + P_2 + P_3 + \ldots = 1$. [3]

(iii) Given that $P_n < 0.001$, show that n satisfies the inequality

$$n > \frac{\log_{10} 0.006}{\log_{10}\left(\frac{5}{6}\right)} + 1.$$

Hence find the least value of n for which $P_n < 0.001$. [4]

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Mark Scheme 4752
June 2007

1	(i) $-\sqrt{3}$ (ii) $\frac{5}{3}\pi$	1 2	Accept any exact form accept $\frac{5\pi}{3}$, $1\frac{2}{3}\pi$. M1 π rad = 180° used correctly	3
2	$y' = 6\times\frac{3}{2}x^{\frac{1}{2}}$ or $9x^{\frac{1}{2}}$ o.e. $y'' = \frac{9}{2}x^{-\frac{1}{2}}$ o.e. $\sqrt{36} = 6$ used interim step to obtain $\frac{3}{4}$	2 1 M1 A1	1 if one error in coeff or power, or extra term f.t. their y′ only if fractional power f.t. their y″ www answer given	5
3	(i) $y = 2\mathrm{f}(x)$ (ii) $y = \mathrm{f}(x - 3)$	2 2	1 if 'y=' omitted [penalise only once] M1 for y = kf(x), k > 0 M1 for $y = \mathrm{f}(x + 3)$ or y = f(x – k)	4
4	(i) 11 27 or ft from their 11 (ii) 20	1 1 2	 M1 for 1 × 2 + 2× 3 + 3 × 4 soi, or 2,6,12 identified, or for substituting n = 3 in standard formulae	4
5	$\theta = 0.72$ o.e 13.6 [cm]	2 3	M1 for 9 = ½ × 25 × θ No marks for using degrees unless attempt to convert B2 ft for 10 + 5 × their θ or for 3.6 found or M1 for $s = 5\theta$ soi	5
6	(i) $\log_a 1 = 0$, $\log_a a = 1$ (ii) showing both sides equivalent	1+1 3	NB, if not identified, accept only in this order M1 for correct use of 3rd law and M1 for correct use of 1st or 2nd law. Completion www A1. Condone omission of *a*.	5
7	(i) curve with increasing gradient any curve through (0, 1) marked (ii) 2.73	G1 G1 3	correct shape in both quadrants M1 for $x \log 3 = \log 20$ (or $x=\log_3 20$) and M1 for $x = \log 20 \div \log 3$ or B2 for other versions of 2.726833.. or B1 for other answer 2.7 to 2.8	5
8	(i) $2(1 - \sin^2\theta) + 7\sin\theta = 5$ (ii) $(2\sin\theta - 1)(\sin\theta - 3)$ $\sin\theta = ½$ 30° and 150°	1 M1 DM1 A1 A1	for $\cos^2\theta + \sin^2\theta = 1$ o.e. used 1st and 3rd terms in expansion correct f.t. factors B1,B1 for each solution obtained by any valid method, ignore extra solns outside range, 30°, 150° plus extra soln(s) scores 1	5

9	i	$y' = 6x^2 - 18x + 12$ $= 12$ $y = 7$ when $x = 3$ tgt is $y - 7 = 12(x - 3)$ verifying $(-1, -41)$ on tgt	M1 M1 B1 M1 A1	condone one error subst of $x = 3$ in their y' f.t. their y and y' or B2 for showing line joining (3, 7) and (−1, −41) has gradient 12	5
	ii	$y' = 0$ soi quadratic with 3 terms $x = 1$ or 2 $y = 3$ or 2	M1 M1 A1 A1	Their y' Any valid attempt at solution or A1 for (1, 3) and A1 for (2,2) marking to benefit of candidate	4
	iii	cubic curve correct orientation touching x- axis only at (0.2,0) max and min correct curve crossing y axis only at −2	G1 G1 G1	 f.t.	3
10	i	970 [m]	4	M3 for attempt at trap rule ½×10×(28+22+2[19+14+11+12+16]) M2 with 1 error, M1 with 2 errors. Or M3 for 6 correct trapezia, M2 for 4 correct trapezia, M1 for 2 correct trapezia.	4
	ii	concave curve or line of traps is above curve (19+14+11 +11+12+16) × 10 830 to 880 incl.[m]	1 M1 A1	Accept suitable sketch M1 for 3 or more rectangles with values from curve.	3
	iii	$t = 10$, $v_{\text{model}} = 19.5$ difference = 0.5 compared with 3% of 19 = 0.57	B1 B1f.t.	or $\frac{0.5}{19} \times 100 \approx 2.6$	2
	iv	$28t - ½t^2 + 0.005t^3$ o.e. value at 60 [− value at 0] 960	M1 M1 A1	2 terms correct, ignore + c ft from integrated attempt with 3 terms	3
11	ai	13	1		1
	aii	120	2	M1 for attempt at AP formula ft their a, d or for 3 + 5 + …+ 21	2
	bi	$\frac{125}{1296}$	2	M1 for $\frac{1}{6} \times \left(\frac{5}{6}\right)^3$	2
	ii	$a = 1/6$, $r = 5/6$ s.o.i. $S_\infty = \frac{\frac{1}{6}}{1-\frac{5}{6}}$ o.e.	1+1 1	If not specified, must be in right order	3
	iii	$\left(\frac{5}{6}\right)^{n-1} < 0.006$ $(n-1)\log_{10}\left(\frac{5}{6}\right) < \log_{10} 0.006$ $n - 1 > \frac{\log_{10} 0.006}{\log_{10}\left(\frac{5}{6}\right)}$ $n_{\min} = 30$ Or $\log(1/6) + \log(5/6)^{n-1} < \log 0.001$ $(n-1)\log(5/6) < \log(0.001/(1/6))$	M1 M1 DM1 B1 M1 M1	 condone omission of base, but not brackets NB change of sign must come at correct place	4

4752: Concepts for Advanced Mathematics (C2)

General Comments

There was a full range of achievement, and there were many excellent scripts. However, a significant minority of candidates were evidently not ready for the examination, and scored very few marks. Section B was generally better received than section A. Most candidates set their work out clearly. Nevertheless, many marks were lost by failing to show sufficient detail of the method – simply providing a statement of an answer to a question worth 3 or 4 marks generally scores zero.

Comments on Individual Questions

Section A

1 The majority of candidates struggled with part (i) – most wrote down as many decimal places as they could from their calculator, and scored zero. However, some weak candidates did gain the first mark – evidently because they had a calculator which would deal with surds.
Part (ii) was generally done well, although a number of candidates lost marks by failing to cancel the fraction down to its lowest terms.

2 This question was very well done, with the majority scoring full marks. Some candidates lost the final mark because they failed to show the intermediate step.

3 Both parts defeated many candidates. A good number lost marks through poor notation, such as omitting “y =”. In most of the better scripts full marks was awarded.

4 Many candidates were evidently not familiar with the inductive definition in part (i), and treated it as an algebraic definition. In part (ii), some candidates simply added together the three terms from part (i) (scoring zero), and a good number apparently had no idea what to do. Some candidates correctly identified 2, 6 and 12, but neglected to find the sum.

5 Most candidates used the correct formula to find θ = 0.72 correctly, but often spoiled their answer by writing it as 0.72π, or multiplying by $\frac{180}{\pi}$. Many then recovered to score full marks in the second part. A significant minority made poor use of the formula book and tried to use $\int \frac{1}{2} r^2 d\theta$.

6 Most scored full marks in part (i), although some candidates decided to write their own question, and evaluate (for example) $\log_{10}1$ and $\log_{10}10$. Only those who made it clear that they understood the results for the general case scored full marks here.
Part (ii) defeated the majority. Most scored one mark for correctly applying the third law of logarithms, but were then unable to apply the second law correctly because of the minus sign, or the 2, or both in $-2\log_a \frac{x^3}{4}$.

7 Marks were thrown away in part (i) by only showing one quadrant or failing to show the y-intercept clearly. Many candidates unnecessarily produced a large table of results and an accurate plot on graph paper, thus wasting valuable time. Full marks were awarded for the correct shape in both quadrants and a clear indication that the curve passes through (0,1).
Most scored full marks in part (ii), a small number of candidates scored zero by finding $\sqrt[3]{20}$.

8 Many candidates attempted to manipulate the given expression without using Pythagoras, scoring zero. Those who did use Pythagoras often used poor notation such as $\cos^2 + \sin^2 = 1$. In the second part a significant minority elected to use the quadratic formula, thus throwing away the first two marks. These candidates often stopped at $\sin\theta = ½$, $\sin\theta = 3$, scoring a total of 0. However, many were able to score full marks here, although some made the basic error of leaving their calculator in radian mode and losing the final two marks.

Section B

9 (i) Most candidates found $\frac{dy}{dx}$ correctly, and then the gradient of the curve at x = 3. Many went on to score full marks, but a common error was to find the equation of the line through (-1,-41) with gradient 12, and then show that this passes through (-1,-41).

(ii) Most candidates did very well here, although there were occasional errors in finding the appropriate y-values.

(iii) A good number of candidates did not see the relevance of the work they had done in part (ii), and started again by producing a table of results and an accurate plot on graph paper. This was unnecessary for full marks – a cubic with correct intercepts and turning points indicated scored full marks.

10 (i) Most candidates scored full marks for this part. In a small number of cases candidates lost marks by failing to use the table, and reading values incorrectly from the curve, or by omitting a pair of brackets, or by using the wrong value for h. A few candidates substituted x-values in, all the way through, scoring zero.

(ii) Those who produced a sketch generally obtained the first mark, but using appropriate rectangles defeated many. A common approach was to use values which were not taken from the curve.

(iii) Most obtained the value 19.5 correctly, but then failed to obtain the second mark because they evaluated $\frac{0.5}{19.5} \times 100$. Some candidates obtained t = 10, speed = 16.5 and then seemed to think that the difference of 2.5 was less than 3% of 19. It was surprising that there was usually no attempt to go back and check their work.

(iv) This was done very well, with some candidates scoring full marks here when the rest of the question was inaccessible to them. There were problems with the first term, however, with many writing 28x or 28^2.

11 (a) The vast majority of candidates scored full marks on this part of the question.

(b) Parts (i) and (ii) were very well done, with most scoring full marks. However, part (iii) defeated most – only the best were able to use logarithms correctly, and very few changed the inequality at the appropriate place. Surprisingly few candidates obtained the correct value of n. Many wrote n > 29.06, scoring zero.